Lec 5: 02/02/2010

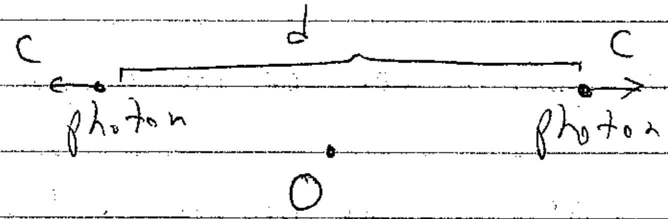
Issues in expanding universe:

We now discuss some common questions, misconceptions, and confusions regarding an expanding universe.

- Consider two points one at the origin $r=0$ and the other at r (assume the universe is flat). The physica[l] distance between the points is $a(t)r$, and the rate at which the distance increases is $\dot{a}(t)r = H\,a(t)r$. At very large r, or for very large $a(t)$, we see that $\dot{a}(t)r > c$. Does this contradict with the fact that no physical signal can travel faster than the speed of light?

The answer is no. $a(t)r$ merely represents the distance between two observers one at $r=0$ and the other at r. The important point is that the

the two observers cannot communicate faster than the speed of light. A simple example where the distance between two points increases faster than c is the following:



According to the observer at O the distance between the two photons moving in opposite directions increases at a speed of 2c. But this does not represent the motion of any physical signal.

Indeed, in an expanding universe the light moves along a null geodesic:

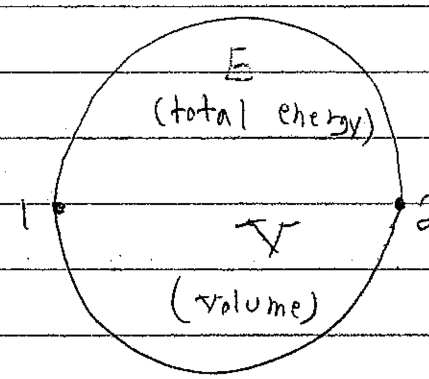
$dt = a_{(t)} dr$

$a_{(t)}\ dr$ is the physical distance travelled by light in a short time interval dt, implying that the physical velocity is 1 (in natural units, where c=1)

What is expanding? Does the distance between every two objects expand? Does every object expand itself? Then, how can we notic expansion as a physical effect?

We knew that the solar system and our galaxy do not expand. But the distance between far away galaxies expand. In fact, if every thing expands there will be no way to figure out the expansion or We then have a "conformal" universe where expansi can be removed from equation by a redefiniti of physical quantities. It is clear that in order to find out about expansion in the distance between two objects, we need a "ruler" that does not expand itself.

As a rule of thumb, Consider two objects:



The volume V of the region between the objects contains a total energy E. We can find the average energy density $\bar{\rho}$ in this region according to

$$\bar{\rho} = \frac{E}{V}$$

Now plug this into the first Friedmann equation:

$$H^2 = \frac{8\pi G}{3}\rho \qquad (\text{lets assume } k=0)$$

H found from this equation represents a

length scale H^{-1} (we use natural units).
If the average density within this distance is the same as $\bar{\rho}$, then everything within this region (including the distance between the two objects) expands at a rate H.
However, if the average density within the distance H^{-1} is $\ll \bar{\rho}$, then the distance between the two objects does not expand.
One can use this rule to see why the milky way does not expand.
Note that gravitationally bound objects do not expand (galaxies, clusters). This has to do with deviation from homogeneity at small scales. In this case we have overdense and underdense regions. Overdense regions

can become bound, and their density can be much larger than the average density in the universe. In a perfectly homogeneous universe, $\rho = \bar{\rho}$ everywhere, and hence the distance between any two objects increases in time.

Now lets discuss some physical effects in an expanding universe.

- Frequency redshift: Consider a photon with frequency ω_1 emitted from a point at radial coordinate r_1 at time t_1, arriving at the origin $r=0$ at time t_2. If the universe is static, the observed frequency ω_2 will be equal to ω_1. However, the situation is different in an expanding universe.

To see this, consider a periodic signal with period $T_1 = \frac{2\pi}{\omega_1}$ sent from a source at r at time T_1. Two successive signals are sent at times t_1 and $t_1 + T_1$. They both travel from r to the origin at the speed of light ("1" in natural units).

For the first signal, the corresponding geodesic obeys the following equation:

$$\int_{r_1}^{0} dr = \int_{t_1}^{t_2} \frac{dt}{a(t)}$$

Similarly, for the second signal, which is sent at time $t_1 + T_1$ and received at time $t_2 + T_2$, we have,

$$\int_{r_1}^{0} dr = \int_{t_1+T_1}^{t_2+T_2} \frac{dt}{a(t)}$$

Since the left hand side of the two equation are equal, we have,

$$\int_{t_1}^{t_2} \frac{dt}{a(t)} = \int_{t_1+T_1}^{t_2+T_2} \frac{dt}{a(t)}$$

If $T_1, T_2 \ll t_2 - t_1$, we have:

$$\int_{t_1+T_1}^{t_2+T_2} \frac{dt}{a(t)} = \int_{t_1}^{t_2} \frac{dt}{a(t)} + \frac{T_2}{a(t_2)} - \frac{T_1}{a(t_1)}$$

Thus,

$$\frac{T_2}{a(t_2)} - \frac{T_1}{a(t_1)} = 0 \Rightarrow \frac{T_2}{T_1} = \frac{a(t_2)}{a(t_1)} \Rightarrow \boxed{\frac{\omega_2}{\omega_1} = \frac{a(t_1)}{a(t_2)}}$$

In an expanding universe $a(t_1) < a(t_2)$, and hence $\omega_2 < \omega_1$, i.e. frequency is redshifted. In a contracting universe, on the other hand, we have a blueshift ($\omega_2 > \omega_1$) since $a(t_1) > a(t_2)$.

The redshift parameter "z" is defined as:

$$\boxed{1+z \equiv \frac{\lambda_2}{\lambda_1} = \frac{a(t_2)}{a(t_1)}}$$

Note that the further the source is, the earlier the photon must take off in order to arrive to the observer at time t_2. This implies that the further the source is, the higher its redshift will be.

- Hubble radius: At a given time t, the inverse of the Hubble expansion rate H represents a length scale H^{-1}. This is called the "Hubble radius" or the "horizon radius." To elucidate, consider a matter dominated universe ($p=0$) or a radiation dominated universe ($p=\frac{1}{3}\rho$).

$$(a)\ p=0 \Rightarrow a_{(t)} = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}} \Rightarrow H = \frac{2}{3t} \Rightarrow \boxed{H^{-1} = \frac{3t}{2}}$$

$$(b)\ p=\frac{1}{3}\rho \Rightarrow a_{(t)} = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}} \Rightarrow H = \frac{1}{2t} \Rightarrow \boxed{H^{-1} = 2t}$$

We see that in these cases H^{-1} is proportional to "t" up to a factor of $O(1)$. But "t" is the distance that light travels from the big bang ($t=0$) up to time "t" (we use natural units where $c=1$). Since no physical signal can propagate faster than the speed of light, Hubble radius represents the maximum distance (up to an $O(1)$ factor) within which the points are in causal contact.

An interesting point is that H^{-1} grows faste than the scale factor $a(t)$ in a matter dominate universe (t vs $t^{\frac{2}{3}}$) or a radiation-dominate universe (t vs $t^{\frac{1}{2}}$). Therefore, any two points in the universe can communicate after a sufficient long time. This has to do with the fact that expansion is decelerating in a matter or radiation dominated universe. For accelerated expansion $a(t)$ grows faster than H^{-1}, and hence two far away points may not come to causal contact ever.